Problem 1.20

Sportscar
A sportscar, Electro-Fiasco I, can accelerate uniformly to 100 km/h in 3.5 s. Its *maximum* braking rate cannot exceed $0.7g$. What is the minimum time required to go 1.0 km, assuming it begins and ends at rest?

Solution

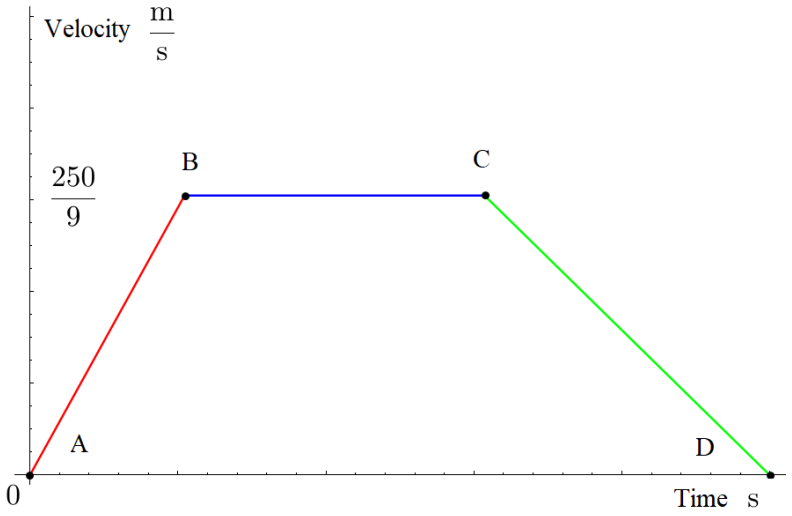
The maximum speed of the sportscar in meters per second is

$$v = 100\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} = \frac{250}{9}\ \frac{\text{m}}{\text{s}},$$

so its uniform acceleration in meters per second squared is

$$a = \frac{\frac{250}{9}\ \frac{\text{m}}{\text{s}}}{3.5\ \text{s}} = \frac{500}{63}\ \frac{\text{m}}{\text{s}^2}.$$

Separate the sportscar's motion into three parts as shown in the figure below.



The Path from A to B

Apply the kinematic formula,

$$x = x_0 + v_0 t + \frac{1}{2}at^2,$$

to the sportscar's path from A to B to find how far the sportscar travels.

$$x_{AB} = 0 + 0 + \frac{1}{2}\left(\frac{500}{63}\right)(3.5)^2 = \frac{875}{18}\ \text{m} \approx 48.6\ \text{m}$$

We conclude that the sportscar travels about 48.6 meters in 3.5 seconds as it accelerates from A to B. Let $t_{AB} = 3.5$ seconds.

The Path from C to D

Apply the kinematic formula,

$$v = v_0 + at,$$

to the sportscar's path from C to D to find how long it takes for the sportscar to come to rest.

$$0 = \frac{250}{9} + (-0.7g)t_{CD}$$

$$(0.7g)t_{CD} = \frac{250}{9}$$

$$t_{CD} = \frac{2500}{63g} \text{ s} \approx 4.05 \text{ s}$$

Apply the kinematic formula,

$$v^2 = v_0^2 + 2a\Delta x,$$

to the sportscar's path from C to D to find how far the sportscar travels before coming to rest.

$$0 = \left(\frac{250}{9}\right)^2 + 2(-0.7g)x_{CD}$$

$$(1.4g)x_{CD} = \left(\frac{250}{9}\right)^2$$

$$x_{CD} = \frac{312500}{567g} \text{ m} \approx 56.2 \text{ m}$$

We conclude that the sportscar travels about 56.2 meters in 4.05 seconds as it decelerates from C to D.

The Path from B to C

Since the whole track is 1000 meters long, the distance the sportscar travels from B to C can be determined.

$$x_{AB} + x_{BC} + x_{CD} = 1000$$

$$\frac{875}{18} + x_{BC} + \frac{312500}{567g} = 1000$$

$$x_{BC} \approx 895.2 \text{ m}$$

Apply the kinematic formula,

$$x = vt,$$

to the sportscar's path from B to C to find how long it takes for the sportscar to reach C.

$$895.2 = \frac{250}{9}t_{BC}$$

$$t_{BC} \approx 32.2 \text{ s}$$

We conclude that the sportscar travels about 895.2 meters in 32.2 seconds as it goes from B to C. Therefore, the minimum time it takes for the sportscar to travel 1 km is

$$t_{AB} + t_{BC} + t_{CD} \approx 39.8 \text{ seconds.}$$